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On Conifolds and D3-branes

Ali Kaya[1]

Center for Theoretical Physics, Texas A& M University,
College Station, Texas 77843, USA.

Abstract

We search for Ricci flat, Kähler geometries which are asymptotic to the cone whose base is the space T^{11} by working out covariantly constant spinor equations. The metrics we find are singular in the interior and introducing parallel D3-branes does not form regular event horizons cloaking the naked singularities. We also work out a supersymmetric ansatz involving only the metric and the 5-form field corresponding to D3-branes wrapping over the non-trivial 2-cycle of T^{11}. We find a system of first-order equations and argue that the solution has an event horizon and the ADM mass per unit volume diverges logarithmically.

[1] ali@rainbow.physics.tamu.edu

The conifold is a 6-dimensional complex manifold described by a quadric equation in C^4

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0. \qquad (1)$$

It can be shown that this quadric is a cone whose base is $S^2 \times S^3$. It is also possible to find a Ricci flat, Kähler metric on the conifold [1], which may be written as

$$ds^2 = dr^2 + r^2 ds^2_{T^{11}}, \qquad (2)$$

where the Einstein space T^{11} has the topology $S^2 \times S^3$. The Einstein metric of T^{11} can be written explicitly

$$ds^2_{T^{11}} = \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin\theta_i^2 d\phi_i^2) + \frac{1}{9}(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i)^2. \qquad (3)$$

The apex of the cone is singular and there are different ways of removing the singularity. It is, for instance, possible to *deform* (1) in such a way that the node is replaced by an S^3. It is also possible to rewrite (1) by a linear change of variables and then make a *resolution*, which replaces the node by S^2. These operations preserve the Calabi-Yau structure of the conifold [1].

Studying N parallel D3-branes placed at the singularity of the conifold [2] one discovers an interesting extension of the AdS/CFT duality [3, 4, 5] where the string theory on $AdS_5 \times T^{11}$ is dual to a certain $\mathcal{N} = 1$ supersymmetric gauge theory [6, 7]. The superconformal field theory has the gauge group $SU(N) \times SU(N)$ and contains chiral superfields with a superpotential. Introducing M fractional [8, 9] D3-branes, which are indeed D5-branes wrapped over the collapsed 2-cycle at the singularity [10], changes the gauge group to $SU(N + M) \times SU(N)$. This theory is no longer conformal, and the relative gauge coupling runs logarithmically [10].

The supergravity solutions in the presence of fractional D-branes has been studied in several papers [10, 11, 12, 13, 14, 15, 16]. It is remarkable that, introducing fractional branes changes the geometry in a controlled way. In the usual D-brane solution the warp factor is the zero eigenvalue of the Laplacian on the transverse space. Introducing fractional D-branes, the differential equation picks up a source term and the the harmonic function is modified so that the warp factor becomes [15]

$$H = 1 + \frac{Q}{r^{7-p}} + h(r), \qquad (4)$$

where $p < 6$ and

$$h(r) \sim \begin{cases} 1/r^{10-2p} & p = 0, 1, 2, 4; \\ \ln r/r^4 & p = 3; \\ \ln r & p = 5. \end{cases} \qquad (5)$$

The case $p = 5$ may be unphysical as discussed in [15].

For $p < 5$, the geometry is asymptotically flat. Removing asymptotically flat region by ignoring the constant term in (4), one can ”zoom in” on the low energy dynamics and decouple the interactions between the supergravity in the bulk and the gauge theory on the branes. For $p = 3$, this gives the gravity dual of the $SU(M + N) \times SU(N)$ gauge theory corresponding M fractional and N regular D3-branes [11]. In this solution, the 3-form flux is responsible for conformal symmetry breaking and indeed the 2-form potential acquires a logarithmic radial dependence which implies the logarithmic running of the gauge couplings in the field theory. As

$r \to \infty$, the solution is regular and can be used as the gravity dual of $SU(N+M) \times SU(N)$ theory in the UV. However, toward small r one encounters a singularity, which implies that the solution should be modified to describe physics in the IR.

On the other hand, for $p = 3$ it is possible to indicate two difficulties in obtaining the gravity dual of the gauge theory from the asymptotically flat solution. The first point is that, due to the special logarithmic correction to the warp factor in (5) the ADM mass per unit volume of the flat solution diverges logarithmically. Therefore, it is indeed hard to consider that solution in the space of physical states of the supergravity theory. The second difficulty is that, the solution does not have an event horizon, again due to the special logarithmic correction. However, as it is well known in the context of AdS/CFT duality, in taking the scaling limit or "zooming in" on the low energy dynamics or decoupling the asymptotically flat region, the presence of an event horizon is responsible for the infinite redshift of the energies and plays the crucial role. Thus, we think that it would be appropriate to consider the gravity dual of $SU(N+M) \times SU(N)$ gauge theory (the background with the warp factor (4) without the constant term) as the scaling limit of some other unknown black-brane solution which has a finite mass and a regular event horizon.

It is also interesting to consider the fate of the naked conifold singularity in the presence of D-branes. It is well known that when parallel D3-branes are placed at the singularity, there forms a regular event horizon cloaking the singularity. Introducing fractional D-branes, the story gets complicated, and more fields play a role in the solution. Naively, one would still expect formation of an event horizon. Alternatively, recalling the fact that the dual gauge theory breaks chiral symmetry in the IR and analyzing the moduli space, one can replace the singular conifold of the supergravity background with the deformed conifold from the beginning, and thus both can solve the singularity problem in the IR and obtain a geometrical realization of chiral symmetry breaking [12]. Finally, it is possible to resolve singularity by adding angular momentum to the supergravity background, which also reduces the number of supersymmetries [17].

Motivated by these recent developments, in this letter we first search for Ricci flat, Kähler geometries asymptotic to the cone whose base is the space T^{11}. These spaces can be viewed as the (singular) deformations or resolutions of the conifold. One may have a purely mathematical interest in finding such metrics having restricted holonomies. However, our main concern here is to understand in the context of supergravity theory how the singularities are modified in the presence of parallel D-branes. As mentioned above, when the D3-branes are located at the singularity of the conifold, there forms an event horizon cloaking the singularity. One may wonder if this is also the case for other singular, Ricci flat, asymptotically conifold metrics. If it is the case, then one would hope to take a scaling or near horizon limit of the solution and obtain gravity duals of certain supersymmetric gauge theories. Unfortunately, the answer turns out to be negative for the spaces we consider; in the presence of D3-branes one still encounters either naked singularities or singular horizons.

In this paper, we also consider a supersymmetric ansatz involving only the metric and the 5-form field corresponding to D3-branes wrapping over the 2-cycle of the space T^{11}. Recalling that wrapped D$p+2$-branes are fractional Dp-branes, the background can be thought to be related to fractional D1-branes. Our ansatz differs from the fractional D1-brane solution of [15] where in addition to self dual 5-form field the dilaton, NS and RR 3-forms acquire non-zero vacuum expectation values. The 2-cycle in our ansatz is a supersymmetric cycle of T^{11} [18], and thus one may claim that the D3-branes can wrap it without exciting other fields. Existence of a supersymmetric background having only the metric and the 5-form field supports this claim. Following [19, 20], we derive a system of first order equations and argue that the ADM mass per unit volume diverges logarithmically and the solution has an event horizon.

Let us consider a 6-dimensional metric of the form

$$\begin{aligned} ds^2 &= f(r)^2 dr^2 + \frac{B(r)^2}{6}(d\theta_1^2 + \sin\theta_1^2 d\phi_1^2) + \frac{C(r)^2}{6}(d\theta_2^2 + \sin\theta_2^2 d\phi_2^2) \\ &+ \frac{D(r)^2}{9}(d\psi + \mathcal{A})^2, \end{aligned} \tag{6}$$

where

$$\mathcal{A} = \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2. \tag{7}$$

Note that $\mathcal{A}$ is the one-form potential of the complex structure[2] on $S^2 \times S^2$. We would like to determine the unknown functions f, B, C and D obeying the boundary conditions

$$f \to 1, \qquad B, C, D \to r \quad \text{as } r \to \infty, \tag{8}$$

so that the metric (6) becomes Ricci flat and Kähler. The boundary conditions make sure that the geometry is asymptotically conic whose base is T^{11}. Instead of calculating Ricci tensor, solving second order, coupled differential equations and further imposing a Kähler structure, we demand existence of a covariantly constant spinor ϵ. It is very well known that this implies Ricci flatness and one can also construct a globally well defined and covariantly constant complex structure

$$J_{ab} = i\,\epsilon^\dagger \Gamma_{ab}\epsilon, \tag{9}$$

obeying

$$J_a{}^b J_b{}^c = -\delta_a^b, \tag{10}$$

where $a, b, c = 1..6$ are tangent space indices on (6). The last equation can be verified by a Fierz identity. In solving the spinor equations, we use the *gauge* covariantly constant spinors on $S^2 \times S^2$ obeying [21]

$$D_\alpha \eta \equiv (\nabla_\alpha + \frac{1}{2}\mathcal{A}_\alpha)\eta = 0 \tag{11}$$

and

$$J_{\beta\alpha}\Gamma^\alpha \eta = i\Gamma_\beta \eta, \tag{12}$$

where the one-form $\mathcal{A}$ is given in (7), $(\alpha, \beta) = 1,..,4$ are tangent space indices and ∇_α is the covariant derivative on $S^2 \times S^2$. One can show that ϵ is a covariantly constant spinor on (6) provided that it is chosen to be a chiral spinor obeying

$$\epsilon = e^{-i/2\psi}\,\eta, \tag{13}$$

and

$$\frac{D'}{fD} = -\frac{D}{B^2} - \frac{D}{C^2} + \frac{3}{D}, \tag{14}$$

$$\frac{B'}{fB} = \frac{D}{B^2}, \tag{15}$$

$$\frac{C'}{fC} = \frac{D}{C^2}, \tag{16}$$

[2]One may consider a more general potential of the form $\mathcal{A} = p\cos\theta_1 d\phi_1 + q\cos\theta_2 d\phi_2$, where p and q are integers. However, it turns out that only when $p = q = 1$ the metric admits covariantly constant spinors.

where $'$ denotes derivative with respect to r. Note that the chirality of ϵ is consistent with (11) and (12). One may surprise by the fact that there are four independent functions and three differential equations. This is simply a manifestation of the reparametrization invariance related to the choice of the coordinate r in the metric (6). One can indeed fix one of the unknown functions by using this invariance as we will do in a moment.

From (15) and (16), one finds that

$$B^2 = C^2 + \gamma^2, \tag{17}$$

where γ is a constant. For $\gamma = 0$ one can fix r-reparametrization invariance by imposing $B = C = r$. Remaining unknown functions f and D can be solved from (14) and (15) to give the following Ricci flat metric[3]

$$ds^2 = \frac{r^6}{r^6 - 1}\, dr^2 + \frac{r^2}{6} \sum_{i=1}^{2} (d\theta_i^2 + \sin\theta_i^2 d\phi_i^2) + \frac{r^2}{9} \left(\frac{r^6 - 1}{r^6}\right) (d\psi + \mathcal{A})^2. \tag{18}$$

Another solution can be obtained by letting $r \to i\,r$ which gives

$$ds^2 = \frac{r^6}{r^6 + 1}\, dr^2 + \frac{r^2}{6} \sum_{i=1}^{2} (d\theta_i^2 + \sin\theta_i^2 d\phi_i^2) + \frac{r^2}{9} \left(\frac{r^6 + 1}{r^6}\right) (d\psi + \mathcal{A})^2. \tag{19}$$

Note that (18) and (19) represent two different geometries, i.e. there is no coordinate transformation that will take (18) into (19).

For $\gamma \neq 0$, we can parametrize B and C by

$$\begin{aligned} B &= \gamma \cosh\rho, \\ C &= \gamma \sinh\rho. \end{aligned} \tag{20}$$

Introducing a new radial coordinate u defined by

$$\frac{D}{f dr} = \frac{1}{du}, \tag{21}$$

and from (20), (15) and (14) we obtain

$$D^2 = \frac{e^{6u}}{\cosh^2\rho \sinh^2\rho} \tag{22}$$

$$e^{6u} = \gamma^2 (\sinh^6\rho + \frac{3}{2}\sinh^4\rho + \mathrm{c}') \tag{23}$$

where c' is a constant. In terms of $r \equiv \gamma \sinh\rho$, this gives the following Ricci flat metric[4]

$$\begin{aligned} ds^2 &= K(r)^{-1}\, dr^2 + \frac{(r^2+\gamma^2)}{6} (d\theta_1^2 + \sin\theta_1^2 d\phi_1^2) + \frac{r^2}{6} (d\theta_2^2 + \sin\theta_2^2 d\phi_2^2) \\ &+ K(r) \frac{r^2}{9} (d\psi + \mathcal{A})^2, \end{aligned} \tag{24}$$

where

$$K(r) = \frac{(r^6 + \frac{3}{2}\gamma^2 r^4 + c)}{r^4 (r^2 + \gamma^2)}, \tag{25}$$

[3]The solution $f = 1$ and $D = r$ corresponds to the conifold.

[4]Note that the coordinate r in (24) is different from the original radial coordinate.

and c is a constant. For $c = 0$, (24) becomes the *resolved* conifold metric which is explicitly given in [13]. Thus here we found that it belongs to a larger two parameter family of metrics (24).

By construction, the metrics (18), (19) and (24) are Ricci flat and admit covariantly constant spinors. Thus the holonomy group of each metric is restricted. Furthermore, one can find a covariantly constant complex structure by using (9). In the tangent space basis $E^r = f dr$, $E^{i_1} = B e^{i_1}$, $E^{i_2} = C e^{i_2}$ and $E^D = D(d\psi + \mathcal{A})$, where e^{i_1} and e^{i_2} refers to tangent space of $S^2 \times S^2$, respectively, the complex structure takes the standard form

$$\begin{aligned} J_{rD} &= -J_{Dr} = 1, \\ J_{i_1 i_1'} &= \epsilon_{i_1 i_1'}, \\ J_{i_2 i_2'} &= \epsilon_{i_2 i_2'}. \end{aligned}$$

One can indeed verify that J_{ab} is covariantly constant, and thus (18), (19) and (24) are Ricci flat, Kähler metrics.

Asymptotically, as $r \to \infty$, all three metrics approach to the conifold (2). All three metrics are also singular in the interior, except the resolved conifold metric which corresponds to $c = 0$ in (24) and is known to be regular. The metric (18) is defined for $r \geq 1$ and as $r \to 1$ $S^2 \times S^2$ has a finite volume but the $U(1)$ bundle parametrized by the coordinate ψ shrinks to zero size forming a singularity. The metrics (19) and (24) are defined for $r \geq 0$. In (19), as $r \to 0$, the $U(1)$ bundle expands (therefore the curvatures decrease) but $S^2 \times S^2$ shrinks to zero size forming a singularity. In (24) and for $c \neq 0$, although one of the S^2's has a finite volume and the $U(1)$ bundle expands, the other S^2 factor shrinks to zero size as $r \to 0$ forming a singularity.

Before introducing D3-branes and studying supergravity solutions, one may be curios about the role played by spheres in the above metrics. Replacing $S^2 \times S^2$ with $R^2 \times R^2$, one may consider a metric of the form.

$$ds^2 = f(r)^2 dr^2 \, + \, B(r)^2 \, (dx^i dx^i) + C(r)^2 \, (dy^i dy^i) \, + \, D(r)^2 (d\psi + \mathcal{A})^2, \tag{26}$$

where

$$\mathcal{A} \; = \; x^i dx^i \, + \, y^i dy^i, \tag{27}$$

$i = 1, 2$ and ψ is not necessarily periodic. This metric represents a line bundle over $R^2 \times R^2$. Working out the covariantly constant spinor equations one finds the following equations

$$\frac{D'}{fD} \; = \; -\frac{D}{2B^2} - \frac{D}{2C^2}, \tag{28}$$

$$\frac{B'}{fB} \; = \; \frac{D}{2B^2}, \tag{29}$$

$$\frac{C'}{fC} \; = \; \frac{D}{2C^2}. \tag{30}$$

Compared to (14)-(16), the only difference (in addition to the one related to normalization of the metric functions) is that the last term in (14) is absent. Nothing that (29) and (30) imply (17), one can again parametrize B and C as in (20) and solve the remaining equations to obtain the following metric

$$ds^2 = 4r^4(r^2+\gamma^2)dr^2 \; + \; r^2 dx^i dx^i \; + \; (r^2+\gamma^2) dy^i dy^i \; + \; \frac{1}{r^2(r^2+\gamma^2)}(d\psi + \mathcal{A}). \tag{31}$$

By construction, this one parameter family of metrics are Ricci flat and Kähler. The structure of (31) is very similar to (24) and can be thought to correspond to the limit where the radius of the spheres blow up. For $\gamma = 0$, (31) has been found in [20], so here we generalize our previous result.

Another possible modification is to replace $S^2 \times S^2$ with a single copy of S^2 and thus consider a four dimensional geometry. Starting with an ansatz of the following form

$$ds^2 = f(r)^2 dr^2 \;+\; B(r)^2\,(d\theta^2 + \sin^2\theta d\phi^2) \;+\; D(r)^2(d\psi + p\cos\theta d\phi)^2, \tag{32}$$

where p is an integer, and using the Killing spinors of S^2, one finds that (32) admits covariantly constant spinors if

$$\frac{D'}{fD} = -\frac{pD}{2B^2}, \tag{33}$$

$$\frac{B'}{fB} = \frac{pD}{2B^2} - \frac{1}{B}. \tag{34}$$

To solve these first order coupled differential equations, we first fix r-reparametrization invariance by imposing $B = r$. After this gauge fixing the non-linear differential equations can be solved exactly which gives the following metric

$$ds^2 = \frac{(1+\sqrt{1+r^2})^2}{(1+r^2)}\,dr^2 \;+\; r^2\,(d\theta^2 + \sin^2\theta d\phi^2) \;+\; \frac{4r^2}{p^2(1+\sqrt{1+r^2})^2}(d\psi + p\cos\theta d\phi)^2. \tag{35}$$

By construction (35) is Ricci flat and Kähler. The metric is regular except at $r = 0$, where there is a conic singularity of the following form

$$\begin{aligned} \text{as}\quad r &\to 0 \\ ds^2 &\to 4dr^2 + r^2\left(d\theta^2 \;+\; \sin^2\theta d\phi^2 \;+\; \frac{1}{p^2}(d\psi + p\cos\theta d\phi)^2\right). \end{aligned} \tag{36}$$

Note that for $p = 1$, last three terms combine to form the standard S^3 metric given in terms of Euler angles.

Now, we would like to introduce parallel D3-branes on spaces (18), (19) and (24), where the branes are located at finite r. We are interested in the fate of the singularities in the presence of D3-branes i.e. if there forms event horizons possibly cloaking the singularities. We assume that the metric and the self-dual 5-form field of IIB theory have the following form

$$\begin{aligned} ds^2 &= \hat{A}(r)^2\, ds_4^2 \;+\; \hat{f}(r)^2 dr^2 \\ &+ \frac{\hat{B}(r)^2}{6}\,(d\theta^2 + \sin\theta^2 d\phi^2) \;+\; \frac{\hat{C}(r)^2}{6}\,(d\theta'^2 + \sin\theta'^2 d\phi'^2) \;+\; \frac{\hat{D}(r)^2}{9}(d\psi + \mathcal{A})^2, \end{aligned} \tag{37}$$

$$F \sim (1 + *)\;\Omega_2 \wedge \Omega_{2'} \wedge (d\psi + \mathcal{A}), \tag{38}$$

where $\Omega_2 \wedge \Omega_{2'}$ is the volume form on $S^2 \times S^2$ with angular coordinates (θ, ϕ), (θ', ϕ'), respectively, $\mathcal{A}$ is given in (7) and ds_4^2 is the metric on the flat 4-dimensional world-volume. This is indeed a natural ansatz to consider, since the solution corresponding to parallel D3-branes located at the singularity of the conifold has this form. Note that, $dF = 0$ and all but the Einstein equations are satisfied. To find the unknown functions, we demand the existence of a Killing spinor on the background which would then imply Einstein equations as shown in [19, 20]. It is not hard to see that the Killing spinor equations are satisfied if one chooses the spinor to be a function of r times the covariantly constant spinor on 6-dimensional transverse Kähler space and

$$\frac{\hat{D}'}{\hat{f}\hat{D}} = -\frac{\hat{D}}{\hat{B}^2} - \frac{\hat{D}}{\hat{C}^2} + \frac{3}{\hat{D}} - \frac{q}{\hat{B}^2\hat{C}^2\hat{D}} \tag{39}$$

$$\frac{\hat{B}'}{\hat{f}\hat{B}} = \frac{\hat{D}}{\hat{B}^2} - \frac{q}{\hat{B}^2\hat{C}^2\hat{D}}, \tag{40}$$

$$\frac{\hat{C}'}{\hat{f}\hat{C}} = \frac{\hat{D}}{\hat{C}^2} - \frac{q}{\hat{B}^2\hat{C}^2\hat{D}}, \tag{41}$$

$$\frac{\hat{A}}{\hat{f}\hat{A}} = \frac{q}{\hat{B}^2\hat{C}^2\hat{D}}, \tag{42}$$

where q is proportional to the dyonic charge of the D3-branes. Although the coupled differential equations seem to be complicated, a simple solution can be found

$$\hat{A} = H^{-1/4}, \quad \hat{f} = H^{1/4}f, \quad \hat{B} = H^{1/4}B, \quad \hat{C} = H^{1/4}C, \quad \hat{D} = H^{1/4}, \tag{43}$$

where f, B, C and D obey (14), (15) and (16), and

$$H' = -\frac{4qf}{DB^2C^2}. \tag{44}$$

Therefore, introducing parallel D3-branes the background still preserves some fraction of supersymmetry of the vacuum, and the geometry is changed by a warp factor obeying (44). It is not very surprising that there is a solution obeying (43) and (44), since it is well known that given a Ricci flat 6-dimensional space one can construct the generalization of the D3-brane solution where the the Ricci flat space plays the role of the transverse space and the warp factor is a harmonic function on it. It is easy to see that H is indeed harmonic on (6).

The solution to (44) can be written as

$$H = 1 + \int_r^\infty \frac{4qf}{DB^2C^2}dr, \tag{45}$$

so that as $r \to \infty$, $H \to 1$. Specifically $H = 1 + O(1/r^4)$ for large r, which shows that the solution has a finite ADM mass per unit volume and asymptotically the geometry becomes the four dimensional flat world-volume times the space (18), (19) or (24). One can also show that the background support non-zero D3-brane charge which is conserved and equal to ADM mass per unit volume.

From (44), we see that H' is always negative. (Note that the functions f, B, C and D are all positive since they are square roots of the metric components.) Therefore, H monotomically increases as r becomes smaller and smaller. Nothing that $H = 1$ at infinity, an event horizon would finally form if H diverges at some r. However, this does not guarantee the regularity of the event horizon.

We now consider three metrics (18), (19) and (24) separately. From (18), the warp factor can be written as

$$H(r) = 1 + \int_r^\infty \frac{4qr}{r^6 - 1}dr. \tag{46}$$

The integral cannot be evaluated in terms of elementary functions, but the behavior near $r = 1$ can easily be found to be $H \sim -\ln(r-1)$. Since H diverges at $r = 1$ there forms an event horizon, which turns out to be a singular surface. Note that, in (18) there was a singularity located at $r = 1$, and thus introducing parallel D3-branes replaces the naked singularity with a null singular surface.

The warp factor corresponding to (19) becomes

$$H(r) = 1 + \int_r^\infty \frac{4qr}{r^6 + 1} dr. \tag{47}$$

Contrary to the above case, the integral now converges as $r \to 0$, where there is a singularity located in (19). Therefore, introducing D3-branes does not change the presence of the naked singularity in (19).

The warp factor corresponding to (24) is equal to

$$H(r) = 1 + \int_r^\infty \frac{4qr}{r^6 + \frac{3}{2}\gamma^2 r^4 + c} dr. \tag{48}$$

As discussed in [13], for $c = 0$ the above integral can be evaluated exactly. Here we note that as $r \to 0$, H diverges as $H \sim 1/r^2$, therefore there forms an event horizon replacing the naked singularity. However, the event horizon turns out to be a singular surface. For $c \neq 0$, the integral would converge as $r \to 0$, thus introducing D3-branes does not remove the naked singularity nor it does form an event horizon.

Till now, we have only considered parallel D3-branes on the spaces (18), (19) and (24), and determined the fate of the naked singularities. We found that the presence of D3-branes does not necessarily imply formation of an event horizon, or if an event horizon would form it is not necessarily regular. We now would like to consider an ansatz corresponding to D3-branes wrapping the supersymmetric 2-cycle of T^{11}. Recalling that the wrapped D$p+2$-branes are indeed fractional Dp-branes, the ansatz can be thought to be related to fractional D1-branes. We will comment on this later. For now let us consider an ansatz of the form,

$$\begin{aligned} ds^2 &= E^2(-dt^2 + dx_1^2) + A^2(dx_2^2 + dx_3^2) \\ &+ \frac{B^2}{6}(d\theta^2 + \sin\theta^2 d\phi^2) + \frac{C^2}{6}(d\theta'^2 + \sin\theta'^2 d\phi'^2) + \frac{D^2}{9}(d\psi + \mathcal{A})^2, \\ F &\sim (1+*)\, dx_2 \wedge dx_3 \wedge (\Omega_2 - \Omega_{2'}) \wedge (d\psi + \mathcal{A}), \end{aligned} \tag{49}$$

where $\Omega_2 \wedge \Omega_{2'}$ is the volume form on $S^2 \times S^2$ with angular coordinates (θ, ϕ), (θ', ϕ'), respectively, and the metric functions E, A, B, C and D depend only on r. It is easy to see that $dF = 0$ and all but Einstein equations of IIB theory are satisfied. The structure of the 5-form field in (49) indicates that the D3-branes wrap over the 2-cycle of T^{11} which is dual[5] to $(\Omega_2 + \Omega_{2'})$. The coordinates t and x_1 span the remaining two dimensions of the D3-brane world-volume, which can be thought to correspond (fractional) D1-branes. The coordinates x_2, x_3 and r together with the 3-cycle of T^{11} dual to the three form $(\Omega_2 - \Omega_{2'}) \wedge (d\psi + A)$ can be identified as the 6-dimensional transverse space.

The background has Killing spinors, and thus obey Einstein equations, provided

$$\frac{D'}{fD} = -\frac{D}{B^2} - \frac{D}{C^2} + \frac{3}{D} - \frac{q}{A^2 B^2 D} + \frac{q}{A^2 C^2 D}, \tag{50}$$

[5]The duality between the finite dimensional vector spaces spanned by the cycles (C_i) and the forms (ω_i), which are the basis of homology and co-homology respectively, is defined with respect to the cup product $\int_C \omega$.

$$\frac{B'}{fB} = \frac{D}{B^2} + \frac{q}{A^2B^2D} + \frac{q}{A^2C^2D}, \tag{51}$$

$$\frac{C'}{fC} = \frac{D}{C^2} - \frac{q}{A^2B^2D} - \frac{q}{A^2C^2D}, \tag{52}$$

$$\frac{A'}{fA} = -\frac{q}{A^2B^2D} + \frac{q}{A^2C^2D}, \tag{53}$$

$$\frac{E'}{fE} = \frac{q}{A^2B^2D} - \frac{q}{A^2C^2D}, \tag{54}$$

where q is proportional to the D3-brane charge. We demand that the metric functions obey the boundary conditions

$$f, A, E \to 1, \qquad B, C, D \to r \quad \text{as } r \to \infty. \tag{55}$$

We could not succeed in solving these equations explicitly. In principle, one can find a perturbative power series solution around flat space, which would determine the asymptotic behavior of the metric. On the other hand, the fact that we found a system of first order equations replacing the second order Einstein equations, would help one to extract some useful information. Indeed, we will argue that the background has an event horizon thus represents black (fractional) D1-branes.

Linearizing the differential equations around flat space, and fixing r-reprametrization invariance by imposing $f = 1$, one finds that the wrapped D3-branes induce following terms as $r \to \infty$

$$A, E = 1 + q\, O(\frac{\ln r}{r^4}), \qquad B, C = r[1 + q\, O(\frac{\ln r}{r^2})], \qquad D = r[1 + q\, O(\frac{\ln r}{r^4})]. \tag{56}$$

Recalling that the two of the transverse directions (corresponding to coordinates x_2 and x_3) are smeared in (49) and thus the real transverse space is four-dimensional, we see from (56) that the solution supports a logarithmically divergent ADM mass per unit volume proportional to q. As noted in the introduction, a similar logarithmic divergence is encountered for fractional D3-branes.

By fixing r-reparametrization invariance in a suitable way, one can also argue that the solution has an event horizon. Imposing

$$f = \frac{4DB^2C^2}{\hat{r}^5(C^2 - B^2)}, \tag{57}$$

where $\hat{r}$ is a radial coordinate, and from (53), one finds that

$$A^2 = 1 + \frac{2q}{\hat{r}^4}. \tag{58}$$

On the other hand (53) and (54) implies $AE = 1$, so

$$E^2 = \left(1 + \frac{2q}{\hat{r}^4}\right)^{-1}. \tag{59}$$

Therefore, as $\hat{r} \to 0$, $E \to 0$, which indicates that there forms an event horizon at $\hat{r} = 0$. Note that the coordinate $\hat{r}$ is different than the coordinate r in (56). Indeed, one can see from (57) that f fails to approach 1, as $\hat{r} \to \infty$. On the other hand, the fact that as $\hat{r} \to \infty$ $A, E \to 1$ indicates that $\hat{r}$ is also a suitable radial coordinate such that the asymptotic region corresponds to large $\hat{r}$.

Is this background related to fractional D1-branes? For now, it is hard to answer this question, since the solution is not known explicitly. Recall that fractional D1-branes are D3-branes wrapped over the 2 cycle of T^{11} *collapsed* at the conical singularity. Therefore, to argue that the above background corresponds to fractional D1-branes we need to know the explicit charge distribution which would give the location of the wrapped D3-branes. Since the 2-cycle in the solution is supersymmetric, one may claim that the wrapped D3-branes can be placed at any radial coordinate. However, when the 2-cycle collapses the curvatures diverge, the (semi-classical) energy of the wrapped D3-branes vanishes and thus extra massless modes appear [18] which indicates that the supergravity description brakes down. (On the other hand, note that the the energy of the collapsed D3-branes diverges logarithmically after integrating out the massless modes [18]. The fact that the ADM mass of the gravity background diverges logarithmically indicates that supergravity still encodes some information about collapsed D3-branes.) In the case of parallel D3-branes placed at the conifold singularity, the energy of the D3-branes does not vanish since they do not wrap over any cycles, and thus effectively they are point-like objects having no internal excitation or energy on T^{11}. Thus parallel D3-branes do not give rise to extra massless modes. In addition, the curvature singularity associated with the conifold is cloaked by the event horizon justifying supergravity description.

As mentioned in the introduction, the above background differs from the fractional D1-brane solution of [15] where the dilaton, NS and RR 3-form fields acquire non-zero vacuum expectation values. We believe that the solution of [15] corresponds to the near horizon limit of the background discussed in this letter.

It is very well known that supergravity brakes down when the curvatures become very large. Therefore, it is difficult to have an appropriate physical picture of manifolds with naked curvature singularities in the context of supergravity. One would naturally expect that introducing parallel D-branes (at or before reaching the singularity) there would form an event horizon cloaking the naked singularity. However, the examples studied in this paper show that this is not always the case; in the presence of D-branes one still encounters either naked singularities or singular horizons. Therefore, the situation is not improved in the context of supergravity. Of course, conic singularities are important exceptions to this as in the case of the conifold. However, in general, it seems supergravity does not offer an appropriate description of D-branes on curved spaces.

Note added: After the submission of the present work to the e-print archive, we learned the paper [22] which has some overlap with our paper. We thank A. Tseytlin for pointing this out to us.

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